Reg. No. | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

UG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.

(For those admitted in June 2024 and later)

PROGRAMME AND BRANCH: B.Sc., STATISTICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
II	PART-III	ELECTIVE GENERIC-2	U24ST2A2	REAL ANALYSIS

Date & Session: 03.05.2025/FN **Time: 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions.
CO1	K1	1.	The set of all irrational numbers is ___________. a) Bounded b) Unbounded c) Countable d) Un countable
CO1	K2	2.	The cantor set k is the set of all numbers x in [0,1] which have a ternary expansion without digit ________. a) 0 b) 1 c) -1 d) ∞
CO2	K1	3.	All sub sequences of a convergent sequence of real numbers converges to the __________. a) Digit b) Interval c) Same limit d) contour
CO2	K2	4.	A non-increasing sequence which is not bounded below diverges to ________. a) 0 b) ∞ c) $-\infty$ d) 1
CO3	K1	5.	If $a_1+a_2+......$converges to s, then $a_2+a_3+.....$converges to ______. a) a_1 b) s c) $s-a_1$ d) $s+a_1$
CO3	K2	6.	If $\sum_{n=1}^{\infty}a_n$ converges and $\sum_{n=1}^{\infty}b_n$ diverges then $\sum_{n=1}^{\infty}(a_n+b_n)$ is .... a) Diverges b) Converges c) Oscillatory d) Non decreasing
CO4	K1	7.	If $'f'$is a real valued function on an interval $J\in R$, then $'f'$is non decreasing on J if _________. a) $f(x)<f(y)$ b) $f(x)\le f(y)$ c) $f(y)<f(x)$ d) $f(y)\le f(x)$
CO4	K2	8.	The real valued function f on the interval $J\subset R$is said to be strictly increasing if a) $f(x)<f(y)$ b) $f(x)\le f(y)$ c) $f(y)<f(x)$ d) $f(y)\le f(x)$
CO5	K1	9.	If A is not measure zero, if $B\subset A$and if B is of measure zero, which of the following is not measure zero? a) $a-b$ b) $A\subset B$ c) $B\subset A$ d) $b-a$
CO5	K2	10.	The subset E of $R^1$is said to be measure zero if for each $\in>0$, there exists a ________number of open intervals $I_1,I_2,.....$such that a) Countable b) Un countable c) Infinite d) Real

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K3	11a.	If $A_1, A_2, ...$are countable sets, then prove that $\bigcup_{i=1}^{\infty} A_i$ is countable.
			(OR)
CO1	K3	11b.	If A is any non-empty subset of R, then prove that A has a greatest lower bound in R
CO2	K3	12a.	If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.
			(OR)
CO2	K3	12b.	If $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.
CO3	K4	13a.	Show that the series $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)$ is divergent.
			(OR)
CO3	K4	13b.	If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ converges.
CO4	K4	14a.	If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then prove that $f(x)+g(x)$ has a limit as $x \to a$ and, in fact $\lim_{x\to a}[f(x)+g(x)] = L+M$
			(OR)
CO4	K4	14b.	If f is a monotone function on the open interval (a,b) and if $c \in (a,b)$, then prove that $\lim_{x\to c+} f(x)$ and $\lim_{x\to c-} f(x)$ both exist.
CO5	K5	15a.	Let f be a bounded function on [a,b] then prove that every upper sum for f is greater than or equal to every lower sum for f
			(OR)
CO5	K5	15b.	If $f \in R[a,b]$ and λ is any real number, then prove that $\lambda f \in R[a,b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$

Course Outcome	Bloom's K-level	Q. No.	**SECTION – C (5 X 8 = 40 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K3	16a.	Prove that set $[0,1] = \{x / 0 \le x \le 1\}$ is uncountable
			(OR)
CO1	K3	16b.	If $f: A \to B$ and if $X \subset B, Y \subset B,$ then prove that $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$
CO2	K4	17a.	Prove that a non-decreasing sequence which is bounded above is convergent.
			(OR)
CO2	K4	17b.	Prove that the sequence $\left\{1+\frac{1}{n}\right\}^n$ is convergent.
CO3	K4	18a.	If $\sum_{n=1}^{\infty} a_n$ is a divergent series of positive terms, then prove that there is a sequence of $\{\in_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but for which $\sum_{n=1}^{\infty} \in_n a_n$ still converges.

CO3	K4	18b.	**(OR)** If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers, then prove that (a) $a_1 \geq a_2 \geq .....\geq a_n \geq a_{n+1} \geq ....$and (b) $\lim_{n\to\infty} a_n = 0$ Then the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.
CO4	K5	19a.	If the real valued function f has a derivative at the point $c \in R^1$, then prove that f is continuous at c.
CO4	K5	19b.	**(OR)** Suppose f has a derivative at c and that g has a derivative at $f(c)$. Then show that $\phi = g \circ f$ has a derivative at c and $\phi^1(c) = g^1[f(c)]f^1(c)$
CO5	K5	20a.	Let f be a bounded function on the closed bounded interval [a,b]. Then $f \in R[a,b]$if and only if, for each $\in > 0$.Prove that there exist a subdivision of [a,b] such that $U[f;a] < L[f;\sigma] + \in$
CO5	K5	20b.	**(OR)** If $f+q \in R[a,b]$, $g \in R[a,b]$, then $f+g \in R[a,b]$then prove that $\int_a^b (f+g) = \int_a^b f + \int_a^b g$