Reg. No. | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

UG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.
(For those admitted in June 2024 and later)

PROGRAMME AND BRANCH: B.Sc. STATISTICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
I	PART - III	CORE-2	U24ST102	PROBABILITY THEORY

Date & Session: 24.04.2025/AN **Time: 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** **Answer ALL Questions.**
CO1	K1	1.	The set of all possible outcomes of an experiment is called: a) Sample space b) Event c) Random variable d) Probability
CO1	K2	2.	What is the probability of an event that is certain to happen? a) 0 b) 1 c) 0.5 d) Cannot be determined
CO2	K1	3.	Random variables can be classified into: a) Continuous and discontinuous b) Discrete and continuous c) Independent and dependent d) Positive and negative
CO2	K2	4.	The sum of all probabilities in a probability mass function is: a) 0 b) 1 c) Greater than 1 d) Less than 1
CO3	K1	5.	A two-dimensional random variable involves: a) One random variable b) Two random variables c) Three random variables d) No random variables
CO3	K2	6.	Joint probability mass function is used for: a) Continuous variables b) Discrete variables c) Non-random variables d) Negative values only
CO4	K1	7.	Mathematical expectation is also known as: a) Probability b) Expected value c) Variance d) Standard deviation
CO4	K2	8.	For a continuous random variable, the expected value is calculated using: a) Summation b) Integration c) Subtraction d) Multiplication
CO5	K1	9.	Which of the following is used to calculate moments? a) M.G.F. b) C.G.F. c) P.M.F. d) C.D.F.
CO5	K2	10.	Weak L.L.N. applies to: a) Large sample sizes b) Only small samples c) Only continuous distributions d) Only negative values

<table>
<tr><th>Course Outcome</th><th>Bloom's K-level</th><th>Q. No.</th><th><u>SECTION – B</u> (5 X 5 = 25 Marks)
Answer <u>ALL</u> Questions choosing either (a) or (b)</th></tr>
<tr><td>CO1</td><td>K3</td><td>11a.</td><td>Prove the following: For any two events A and B,
i)$P(\bar{A} \cap B) = P(B) - P(A \cap B)$.
ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$.
(OR)</td></tr>
<tr><td>CO1</td><td>K3</td><td>11b.</td><td>State and prove addition theorem of probability.</td></tr>
<tr><td>CO2</td><td>K3</td><td>12a.</td><td>A random variable X has the following probability function:
<table>
<tr><td>Values of X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P(x)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>k^2</td><td>$2k^2$</td><td>$7k^2$ +k</td></tr>
</table>
i. Find k
ii. Evaluate P(X<6),P(X≥ 6) and P(0<X<5)
(OR)</td></tr>
<tr><td>CO2</td><td>K3</td><td>12b.</td><td>A continuous r,v X has a p.d.f f(x)=3x^2, 0 ≤ x ≤ 1. Find a and b such that
i) P(X≤ a) = $P(X > a)$ ii) P(X>b) =0.05.</td></tr>
<tr><td>CO3</td><td>K4</td><td>13a.</td><td>A random observation on a bi-variate population (X,Y) can yield one of the following pairs of values with probabilities noted against them.
<table>
<tr><th>For each observation pair</th><th>probability</th></tr>
<tr><td>(1,1);(2,1);(3,3);(4,3)</td><td>$\frac{1}{20}$</td></tr>
<tr><td>(3,1);(4,1);(1,2);(2,2);
(3,2);(4,2);(1,3);(2,3)</td><td>$\frac{1}{10}$</td></tr>
</table>
Find the probability that Y=2given that X=4.Also find the probability that Y=2.
OR</td></tr>
<tr><td>CO3</td><td>K4</td><td>13b.</td><td>The joint probability distribution of two random variables X and Y is given by:
P (X=0, Y=1) =$\frac{1}{3}$,P(X=1, Y=-1) =$\frac{1}{3}$, P(X = 1, Y = 1) = $\frac{1}{3}$
Find
i. Marginal distributions of X and Y
ii. The conditional probability distribution of X given Y=1.</td></tr>
<tr><td>CO4</td><td>K4</td><td>14a.</td><td>State and prove addition theorem of Expectation.
(OR)</td></tr>
<tr><td>CO4</td><td>K4</td><td>14b.</td><td>If X is a random variable, then V (aX+b) = a^2V(X), where a and b are constants.</td></tr>
<tr><td>CO5</td><td>K5</td><td>15a.</td><td>Prove that the characteristic function $\varphi_X(t)$ is bounded by 1.
(OR)</td></tr>
<tr><td>CO5</td><td>K5</td><td>15b.</td><td>State and prove the additive property of Cumulants.</td></tr>
</table>

Course Outcome	Bloom's K-level	Q. No.	<u>SECTION – C</u> (5 X 8 = 40 Marks) Answer <u>ALL</u> Questions choosing either (a) or (b)
CO1	K3	16a.	State and prove Boole's Inequality. (**OR)**
CO1	K3	16b.	State and prove Baye's Theorem.

CO2	K4	17a.	Suppose that the time in minutes that a person has to wait at a certain bus stop for a bus is formed to be a random phenomenon, with a probability function specified by the distribution function: $f(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{x}{8} & , \quad 0 \le x < 2 \\ \frac{x^2}{16} & , \quad 2 \le x < 4 \\ 1 & , \quad x \ge 4 \end{cases}$ i. What is the probability that a person will have to wait a. more than 2 minutes b. less than 2 minutes c. between 1&2 minutes ii. What is the conditional probability that the person will have to wait for a bus for a. More than 2 minutes, given that it is more than 1 minute b. Less than 2 minutes given that it is more than 1 minutes. **(OR)**
CO2	K4	17b.	If $f(x) = \begin{cases} \frac{x}{15}, & x = 1,2,3,4,5 \\ 0, & otherwise \end{cases}$ Find i. P(X=1 or 2) ii. $P\{\frac{1}{2} < X < \frac{5}{2} \mid X > 1\}$
CO3	K4	18a.	For the adjoining bivariate probability distribution of X and Y

Y / X	**1**	**2**	**3**	**4**	**5**	**6**
0	**0**	**0**	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	**0**	$\frac{2}{64}$

Find i. P (X≤1, Y=2) ii. P (X≤1) iii. P (Y≤3) iv. P (X<3, Y≤4)

(OR)

CO3	K4	18b.	For the joint probability distribution of two random variables X and Y given below:

			<table><tr><th>Y / X</th><th>1</th><th>2</th><th>3</th><th>4</th><th>Total</th></tr><tr><td>1</td><td>$\frac{4}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{2}{36}$</td><td>$\frac{1}{36}$</td><td>$\frac{10}{36}$</td></tr><tr><td>2</td><td>$\frac{1}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{2}{36}$</td><td>$\frac{9}{36}$</td></tr><tr><td>3</td><td>$\frac{5}{36}$</td><td>$\frac{1}{36}$</td><td>$\frac{1}{36}$</td><td>$\frac{1}{36}$</td><td>$\frac{8}{36}$</td></tr><tr><td>4</td><td>$\frac{1}{36}$</td><td>$\frac{2}{36}$</td><td>$\frac{1}{36}$</td><td>$\frac{5}{36}$</td><td>$\frac{9}{36}$</td></tr><tr><td>Total</td><td>$\frac{11}{36}$</td><td>$\frac{9}{36}$</td><td>$\frac{7}{36}$</td><td>$\frac{9}{36}$</td><td>1</td></tr></table> Find i. The marginal distributions of X and Y ii. Conditional distribution of X given the value of Y=1.
CO4	K5	19a.	State and prove variance of a linear combination of random Variables. **(OR)**
CO4	K5	19b.	State and prove Cauchy -Schwartz Inequality.
CO5	K5	20a.	State and prove Chebyshev's Inequality. **(OR)**
CO5	K5	20b.	State and prove Bernoulli's law of large numbers.